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On the Computation of the Matrix Logarithm

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Resumo

A matrix $X \in \mathbb{C}^{n\times n}$ is the logarithm of a matrix $A \in \mathbb{C}^{n\times n}$ if and only if $e^X = A$. Equivalently, A has a logarithm if and only if A has no eigenvalues on $\mathbb{R}^-$, the closed negative real axis. If the imaginary parts of the eigenvalues of X lie in the interval $(-\pi, \pi)$, the logarithm, $\log(A)$, is unique and called *the principal logarithm*.

The inverse scaling and squaring method is a popular method for computing the matrix logarithm. It is an extension to matrices of the technique that Briggs used in the 17th century to compute his table of logarithms. The method first computes $A^{1/2^s}$, for an integer s large enough so that $A^{1/2^s}$ is close to the identity, then approximates $\log(A^{1/2^s})$ by $r_m(A^{1/2^s} - I)$, where r_m is an $[m/m]$ Padé approximant to the function $\log(1 + x)$, and finally forms the approximation $\log(A) \approx 2^s r_m(A^{1/2^s} - I)$. This approximation exploits the identity

$$\log(A) = 2^s \log(A^{1/2^s}).$$

In this work we make several improvements to the method. We introduce backward error analysis to replace the previous forward error analysis; obtain backward error bounds in terms of the quantities $\|A^p\|^{1/p}$, for several small integer p, instead of $\|A\|$, on which the existing algorithms are based; and use special techniques to compute the argument of the Padé approximant more accurately. We derive one algorithm that employs a Schur decomposition, and thereby works with triangular matrices, and another that requires only matrix multiplications and the solution of multiple right-hand side linear systems. Numerical experiments show the new algorithms to be generally faster and more accurate than their existing counterparts and suggest that the Schur-based method is the method of choice for computing the matrix logarithm.